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Let $N' = c/d$. We shall show that we may express c/d as

$$\frac{1}{l} + \frac{c_1}{d_1},$$

when l is $> r$ (else would c/d be $> 1/r$), c_1/d_1 is $< 1/l$, and $c_1 < c$. If d is divisible by c , the thing is done ($c_1 = 0$). Otherwise, let

$$d = (l - 1)c + m, \quad 0 < m < c.$$

Then

$$\frac{c}{d} = \frac{1}{l} + \frac{c - m}{dl}$$

is an expression of the kind desired, since $(c - m)/dl < c/dl < 1/l$ (for c/d is a proper fraction).

Now apply the same process to c_1/d_1 , which is $< 1/l$:

$$\frac{c_1}{d_1} = \frac{1}{l_1} + \frac{c_2}{d_2}, \quad l_1 > l.$$

Continuing in this way, we shall eventually get for N' an expression of the kind we are seeking:

$$N' = \frac{c}{d} = \frac{1}{l} + \frac{1}{l_1} + \cdots + \frac{1}{l_s},$$

where

$$r < l < l_1 < \cdots < l_s;$$

for the process must come to a stop since the numerators $c_1, c_2, \cdots$ continually decrease (the process being accelerated whenever one of the fractions c_i/d_i reduces).

The procedure may be described in a few words: to keep adding to our set of fractions the largest one available (*i. e.*, one that has not already been used and whose denominator is not $< m$) that will not make their sum greater than the given rational number.

It is, perhaps, interesting to illustrate the methods employed by a numerical example. Let it be required to express $7/22$ as $\sum_{n=10}^{n=10+k} \frac{a_n}{n}$.

$$\frac{7}{22} = \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \frac{29}{660}, \quad \text{where} \quad \frac{29}{660} < \frac{1}{13};$$

$$\frac{29}{660} = \frac{1}{23} + \frac{7}{15180}; \quad \frac{7}{15180} = \frac{1}{2169} + \frac{3}{32925420} = \frac{1}{2169} + \frac{1}{10975140};$$

the numerators 29, 7, 3 forming a decreasing series, and the last "remainder," $3/32925420$, reducing to lower terms.

Our result then is:

$$\frac{7}{22} = \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \frac{1}{23} + \frac{1}{2169} + \frac{1}{10975140}.$$

QUESTIONS AND DISCUSSIONS.

EDITED BY U. G. MITCHELL, University of Kansas.

In April, 1915, there was published in this department of the MONTHLY (Vol. XXII, pp. 119-121) a summary of remarks made by Commissioner Snedden of Massachusetts at the Cincinnati meeting of the N. E. A., Feb. 25, 1915, on the topic "Can algebra and geometry be reorganized so as to justify their retention for high-school pupils not likely to enter technical schools?" These remarks

were published to call the attention of teachers of college mathematics to a movement having for its purpose the practical elimination of algebra and geometry from the secondary-school curriculum. A question was asked in connection with the summary and the replies received were so few as to indicate that the large majority of our readers are comparatively indifferent to the movement. They may have thought that it was a matter which interested high-school teachers only.

The movement is, however, one of vital concern to teachers of college mathematics for at least two reasons. In the first place, it will not be long until the same attack will be made upon the teaching of college mathematics in other than technical schools. A revaluation and reorganization of college mathematics will be demanded, and teachers of college mathematics will do well to take some observations of the movement now in order that they may the better understand the situation when it confronts them. In the second place, if this movement is successful the teacher of college mathematics must begin his teaching on a foundation including little beyond the four fundamental operations in arithmetic.

The reader who inclines to the view that these possibilities are too remote for serious consideration will do well to read "A Modern School," by Abraham Flexner, issued by the General Education Board (61 Broadway, New York) as Occasional Paper No. 3 (23 pp.), and sent gratis on request.

The following extract from an editorial in the *New Republic* for April 8, 1916 (pp. 249-250), is doubtless somewhat typical of the way Dr. Flexner's paper will be taken by editors of other papers, especially by editors of the more radical educational journals:

Dr. Abraham Flexner's manifesto of a modern school, issued this week by the General Education Board, cuts out at a blow all the tangled controversy of schoolmen over what shall be taught and why. The modern school would include nothing in its curriculum for which an affirmative case can not now be made out. Grammar, formal history, dead languages, formal mathematics, are in school now because of tradition and assumption. It is useless to ask whether a knowledge of these subjects is valuable. The fact is that children today do not get value from them. That is all a modern school need know. . . . Dr. Flexner's manifesto puts the whole problem as concisely as it can be put. He gives us a standard, upon which progressive educators already agree, and which can be used to test and to judge the individual school. If the teacher is to have an educational creed, this is the creed for the American school of the immediate morrow.

Dr. Flexner's guiding thesis and its bearing on mathematics are thus stated by him (p. 17):

Modern education will include nothing simply because tradition recommends it or because its inutility has not been conclusively established. It proceeds in precisely the opposite way: *it includes nothing for which an affirmative case can not now be made out.* As has already been intimated, this method of approach would probably result in greatly reducing the time allowed to mathematics, and decidedly changing the form of what is still retained. If, for example, only so much arithmetic is taught as people actually have occasion to use, the subject will sink to modest proportions; and if this reduced amount is taught so as to serve real purposes, the teachers of science, industry and domestic economy will do much of it incidentally. The same policy may be employed in dealing with algebra and geometry. What is taught, when it is taught, and how it is taught will in that event depend altogether on what is needed, when it is needed, and the form in which it is needed.

To those of us who have been accustomed to think of mathematics as the simplest form of abstract reasoning it will seem a bit difficult to reconcile the foregoing with the following statement found on pp. 9-10:

But in the end, if the Modern School is to be adequate to the need of modern life, this concrete training must produce sheer intellectual power. Abstract thinking has perhaps never played so important a part in life as in this materialistic and scientific world of ours—this world of railroads, automobiles, wireless telegraphy, and international relationships.

Dr. Snedden, Dr. Flexner and others who are attacking mathematics are insisting that the burden of proof to show that it should remain in the curriculum is upon those who advocate its continuance and not upon those who would eliminate it. To the writer it seems that these iconoclasts are placing entirely too much faith in the methods of experimental pedagogy so enthusiastically advocated in schools of education at the present time. Experimental pedagogy is a great improvement, but it should be borne in mind that it is only an improvement. It merely attempts to formulate explicitly experimentation which the race has long carried on informally. Humanity decided from its experience that mathematics was valuable and placed it in the course of study. It would seem that the burden of proof ought to be upon the man who would replace it with something else to demonstrate beyond question that the substitute is better than the original.

If these iconoclasts are right, the college teachers ought to be among the first to discover it and to aid in making proper readjustments. But regardless of whether they are right or wrong, teachers of college mathematics ought to be giving more thought and attention to the purposes of their teaching and the sequence and content of courses. Much remains to be done toward organizing and standardizing the undergraduate mathematical curriculum in colleges and universities. There is a somewhat general agreement as to sequence and content of courses so far as to include elementary differential and integral calculus; but beyond that point there is great variation both in opinion and practice.

It ought not to be that each college or university should be left to work out these problems by itself. There ought to be such coöperation as would enable each school to profit by the experience and counsel of others.

The editors would like to see more use made of this department in attacking these and such kindred problems as the proper differentiation of courses for groups of students specializing along different lines. Questions 2, 7, 11, 16, 27, 30 and 31 serve as a beginning in this direction. We hope that more questions will be sent in and also many more replies to questions 27, 30 and 31.

DISCUSSIONS.

RELATING TO "THE UNIFICATION OF FRESHMAN MATHEMATICS."

BY F. L. GRIFFIN, Reed College, Portland, Ore.

Agreeing heartily with nearly all that Mr. Nyberg says in describing his very interesting and valuable course, in the April issue of the MONTHLY, I reluctantly feel obliged to record a difference of opinion on certain points.

- (1) *As to the comparative difficulty of his course and the one which I outlined.*